

5.1.B6b

Řešte soustavu lineárních rovnic (nad \mathbb{R}), v závislosti na parametru $a \in \mathbb{R}$:

$$\begin{aligned} ax_1 - 4x_2 + 9x_3 + 10x_4 &= 11 \\ 2x_1 - x_2 + 3x_3 + 4x_4 &= 5 \\ 4x_1 - 2x_2 + 5x_3 + 6x_4 &= 7 \\ 6x_1 - 3x_2 + 7x_3 + 8x_4 &= 9 \end{aligned}$$

$$\begin{array}{l} (2) \\ (3) \\ (4) \\ (1) \end{array} \left(\begin{array}{cccc|c} 2 & -1 & 3 & 4 & 5 \\ 4 & -2 & 5 & 6 & 7 \\ 6 & -3 & 7 & 8 & 9 \\ a & -4 & 9 & 10 & 11 \end{array} \right) \sim \begin{array}{l} (1) \\ -2 \cdot (1) + (2) \\ -3 \cdot (1) + (3) \\ -\frac{a}{2} \cdot (1) + (4) \end{array} \left(\begin{array}{cccc|c} 2 & -1 & 3 & 4 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & -2 & -4 & -6 \\ 0 & -4 + \frac{a}{2} & 9 - \frac{3a}{2} & 10 - 2a & 11 - \frac{5a}{2} \end{array} \right) \stackrel{(3)=2 \cdot (2)}{\sim}$$

$$\begin{array}{l} (1) \\ 2 \cdot (4) \\ -(2) \end{array} \left(\begin{array}{cccc|c} 2 & -1 & 3 & 4 & 5 \\ 0 & a-8 & 18-3a & 20-4a & 22-5a \\ 0 & 0 & 1 & 2 & 3 \end{array} \right)$$

i) $a = 8$

$$\left(\begin{array}{cccc|c} 2 & -1 & 3 & 4 & 5 \\ 0 & 0 & -6 & -12 & -18 \\ 0 & 0 & 1 & 2 & 3 \end{array} \right) \stackrel{(2)=-6 \cdot (3)}{\sim} \left(\begin{array}{cccc|c} 2 & -1 & 3 & 4 & 5 \\ 0 & 0 & 1 & 2 & 3 \end{array} \right)$$

$$x_4 = k$$

$$x_3 = -2x_4 + 3 = 3 - 2k$$

$$x_1 = l$$

$$x_2 = 2x_1 + 3x_3 + 4x_4 - 5 = 2l + 3(3 - 2k) + 4k - 5 = 4 + 2l - 2k$$

$$K_1 = \{(l; 4 + 2l - 2k; 3 - 2k; k) | k, l \in \mathbb{R} \text{ lib.}\}$$

ii) $a \in \mathbb{R} - \{8\}$

$$x_4 = k$$

$$x_3 = -2x_4 + 3 = 3 - 2k$$

$$(a - 8)x_2 = (3a - 18)x_3 + (4a - 20)x_4 + 22 - 5a = (3a - 18)(3 - 2k) + (4a - 20)k + 22 - 5a$$

$$= 9a - 6ak + 36k - 54 + 4ak - 20k + 22 - 5a = 4a - 2ak + 16k - 32$$

$$x_2 = \frac{4a - 2ak + 16k - 32}{a - 8} = 4 - 2k$$

$$2x_1 = x_2 - 3x_3 - 4x_4 + 5 = 4 - 2k - 3(3 - 2k) - 4k + 5 = 0 \Rightarrow x_1 = 0$$

$$K_2 = \{(0; 4 - 2k; 3 - 2k; k) | k \in \mathbb{R} \text{ lib.}\}$$